Multiphonon and "hot"-phonon Isovector Electric-Dipole Excitations

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We argue that a substantial increase in the cross section for Coulomb excitation in the region of the Double Giant Dipole Resonance should be expected from Coulomb excitation of excited states involved in the spreading of the one-phonon resonance, in a manifestation of the Brink-Axel phenomenon. This generates an additional fluctuating amplitude and a corresponding new term to be added incoherently to the usual cross-section. The appropriate extension of an applicable reaction calculation is considered in order to estimate this effect.

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The Coulomb excitation of two-phonon giant resonances at intermediate energies has generated considerable interest in the last few years [1]. The isovector double giant dipole resonance (DGDR) has been observed in ^{136}Xe [2], ^{197}Au [3], and ^{208}Pb [4–6]. The isoscalar double giant quadrupole resonance has also been observed in the proton emission spectrum from the collision of ^{40}Ca with ^{40}Ca at a laboratory energy of 44 A Mev [7].When the data on DGDR excitation for ^{136}Xe and ^{197} Au are compared with coupled-channel Coulomb excitation calculations [8], it is found that, in the harmonic approximation, the calculated cross sections are a factor of 2 to 3 smaller than the measured ones. A similar discrepancy, albeit somewhat smaller, is found for ^{208}Pb.

Several effects that are not taken into account in the coupled-channel theory have been considered as possible explanations of this discrepancy. As examples, we mention the effect of anharmonicities [9,10] and the quenching of the 1^+ DGDR state [11]. Here we will consider a new potentially important mechanism, which consists in the (one phonon) Coulomb excitation of backgrount states responsible for the large spreading width of the one-phonon GDR, as suggested long ago by Brink and Axel [12]. Due to the complicated background of intrinsic states, the amplitude for this process varies rapidly wth energy and possesses an average close to zero. Its contribution to the cross section can be sizable, however. In close analogy to this situation is the the well-known case of nucleon-nucleus elastic scattering. There, the cross section is the sum of the slowly-varying contribution of average optical scattering and of the fluctuating contribution compound nucleus formation and decay. In figure 1 we show a schematic picture of the couplings involved.

We first sumarize our main result. The cross-section for Coulomb excitation to the DGDR energy region contains in fact two distinct components which peak at $\sim 2E_{GDR}$. However, while the usual component σ_{DGDR} has a width which may be estimated as $\sim 2\Gamma_{GDR}$, the fluctuating Brink-Axel component has a width which is just $\sim \Gamma_{GDR}$. As a result of this, the bump observed in the two-phonon region has an effective width between these limits. The enhancement factor for the peak-value of the cross-section will be given roughly by $(1+\Gamma_1^{\downarrow}/\Gamma_1)$, whereas the cross-section integrated over the peak is just about $(1+\frac{1}{2}\Gamma_1^{\downarrow}/\Gamma_1)\sigma_{DGDR}$ due to the smaller width of the second component. For heavy nuclei $\Gamma_1^{\downarrow} \sim \Gamma_1$, and we get enhancement factors of ~ 2 and $\sim 3/2$ for the peak and for the integrated cross-sections respectively. Furthermore, these enhancement factors should be reduced and tend to unity as the collision time becomes shorter than the inverse GDR width, at higher bombarding energies. A simple model illustrating these features will be discussed in what follows.

Giant resonances have many characteristics that suggest a treatment in terms of simple collective degrees of freedom. The first and foremost of these is their classical interpretation in terms of macroscopic shape oscillations of the nucleus. The properties of multiple excitations of these resonances would then suggest that they are simple bosonic degrees of freedom. The Brink-Axel hypothesis, which assumes that a giant dipole resonance may be constructed on each of the intrinsic excited states of the nucleus, suggests that the resonances can be considered as degrees of freedom independent of the intrinsic states. Of course, the microscopic representation of the giant resonances, in terms of of the intrinsic particle-hole states, implies that their treatment as independent degrees of freedom can only be approximate. Yet, in many instances, it seems to be a very good approximation.

As a model of the multiple excitation and decay of a giant resonance, we will consider the excitation of a collective

degree of freedom of a target nucleus by an inert projectile and the subsequent decay of the collective states into complex intrinsic ones. We can loosely follow the development given by Ko years ago [13]. His work was motivated by the need to incorporate the collective features of the Copenhagen approach to deeply-inelastic heavy-ion collisions [14] into the Heidelberg description [15] of these processes.

As did Ko, we take the Hamiltonian describing the two colliding nuclei to be

$$H(\vec{r},\alpha,\xi) = h_0(\vec{r}) + h_1(\alpha) + h_2(\xi) + U(\vec{r},\xi) + V(\vec{r},\alpha) + W(\alpha,\xi) \tag{1}$$

where $h_0(\vec{r}) = \vec{p}^2/2m + V_0(\vec{r})$ is the Hamiltonian for the relative motion in the ground state, $h_1(\alpha)$ and $h_2(\xi)$ are the collective and intrinsic Hamiltonians, respectively, $W(\alpha,\xi)$ is the coupling between the collective and the intrinsic degrees of freedom and $U(\vec{r},\xi)$ and $V(\vec{r},\alpha)$ are the couplings between the relative coordinate and the intrinsic and collective degrees of freedom, respectively. As we wish to concentrate our attention on the excitation of the collective states alone through the relative motion, we simplify by taking the direct coupling of the relative motion to the intrinsic states to be null, $U(\vec{r},\xi) = 0$. We write the collective and intrinsic spectra and states as $h_1(\alpha)\,|n\rangle = e_n\,|n\rangle$ and $h_2(\xi)\,|\nu\rangle = \varepsilon_\nu\,|\nu\rangle$. In the case that the collective spectrum represents multiple excitations of a giant resonance, we would expect $e_n \simeq n\,e_1$.

We write the uncoupled scattering states of the relative motion with an incoming or outgoing wave boundary condition corresponding to asymptotic wavenumber $\vec{k}$ as

$$h_0\,\left|\varphi_{\vec{k}}^{\pm}\right\rangle = \left(\frac{\vec{p}^{\,2}}{2m} + V_0\right)\left|\varphi_{\vec{k}}^{\pm}\right\rangle = E_{\vec{k}}\,\left|\varphi_{\vec{k}}^{\pm}\right\rangle. \tag{2}$$

The free Green's function can then be expressed in terms of a diagonal sum over the triple product states as

$$G_0^{+}(E) = \sum_{n,\nu}\int d^3k\,\frac{\left|n,\nu,\varphi_{\vec{k}}^{+}\right\rangle\left\langle n,\nu,\varphi_{\vec{k}}^{+}\right|}{E - E_{\vec{k}} - e_n - \varepsilon_\nu + i\,\eta}. \tag{3}$$

We point out that an equivalent expression for $G_0^{+}(E)$ is obtained by substituting the scattering states satisfying an outgoing wave boundary conditions for the incoming wave states used above.

In this basis, the matrix elements of the couplings take the form

$$\left\langle m,\mu,\varphi_{\vec{k}}^{+}\right| V(\vec{r},\alpha)\left|n,\nu,\varphi_{\vec{k}'}^{+}\right\rangle = \delta_{\mu\nu}\,\left\langle m,\varphi_{\vec{k}}^{+}\right| V\,\left|n,\varphi_{\vec{k}'}^{+}\right\rangle \tag{4}$$

and

$$\left\langle m,\mu,\varphi_{\vec{k}}^{+}\right| W(\alpha,\xi)\left|n,\nu,\varphi_{\vec{k}'}^{+}\right\rangle = \delta^{(3)}(\vec{k}-\vec{k}')\,\,\langle m,\mu|\,W\,|n,\nu\rangle\,. \tag{5}$$

The first of these says that the coupling of the relative motion to the collective degree of freedom due to $V(\vec{r},\alpha)$ does not affect the intrinsic state. The second says that the coupling of the collective degree of freedom to the intrinsic ones due to $W(\alpha,\xi)$ does not affect the relative motion of the projectile and target. The transitions induced by $W(\alpha,\xi)$ are internal to the target.

We assume that the complex intrinsic states are statistical and use a schematic random-matrix model to describe their matrix elements.We take for the first and second moments of the matrix elements $\overline{\langle m,\mu|\,W\,|n,\nu\rangle} = 0$ and

$$\overline{\langle m,\mu|\,W\,|n,\nu\rangle\,\langle n',\nu'|\,W\,|m',\mu'\rangle} = \delta_{mm'}\delta_{nn'}\delta_{\mu\mu'}\delta_{\nu\nu'}\,\overline{|\langle n,\nu|\,W\,|m,\mu\rangle|^2} \tag{6}$$

The statistical hypotheses only require that the average squared matrix elements vanish for $\mu \neq \mu'$ or $\nu \neq \nu'$. For simplicity, we take them to vanish for $m \neq m'$ and $n \neq n'$ as well.

We can now use the statistical hypotheses on the matrix elements of W to calculate the average (optical) Green's function $\overline{G}^{+}(E)$. This has been done by Ko [13] and his result can be written as (see also [16])

$$\overline{G}_{\vec{k}n\nu}^{+}(E) = \frac{1}{E - E_{\vec{k}} - e_n - \varepsilon_\nu + i\,\Gamma_{n\nu}/2} \tag{7}$$

where $\Gamma_{n\nu}$ is the total width of the resonance, comprising an escape width $\Gamma_{n\nu}^{\uparrow}$ plus a spreading width $\Gamma_{n\nu}^{\downarrow}$, viz. $\Gamma_{n\nu} = \Gamma_{n\nu}^{\downarrow} + \Gamma_{n\nu}^{\uparrow}$. The complete expression for the average Green's function is then

$$\overline{G}^{+}(E) = \sum_{n,\nu} \int d^3k \, \frac{\left|n,\nu,\varphi^{+}_{\vec{k}}\right\rangle \left\langle n,\nu,\varphi^{+}_{\vec{k}}\right|}{E - E_{\vec{k}} - e_n - \varepsilon_\nu + i\,\Gamma_{n\nu}/2} \tag{8}$$

It is worthwhile pausing a moment to interpret this expression. With the exception of the ground-state component of the Green's function, all others have a finite width in their denominator, However, the scattering states that enter are still those corresponding to the self-adjoint Hamiltonian h_0. Yet, in contrast to the free Green's function, for which the component corresponding to the collective-intrinsic state $|n,\nu\rangle$ contains only the scattering states $\left|\varphi^{+}_{\vec{k}}\right\rangle$ with energy $E_{\vec{k}} = E - e_n - \varepsilon_\nu$, the component of the average Green's function contains an envelope of scattering states about this energy, with the extent of the envelope determined by the width $\Gamma_{n\nu}$. The relative phases of the contributions to this envelope are such that the outgoing waves of $\overline{G}^{+}$ are decaying waves, as can easily be verified by evaluating the integral as a contour integral in the complex plane.

We note that at low excitation energies, the widths of the $|1,\nu\rangle$ states are dominated by their spreading widths, as the contributions to the escape widths, from both the collective state and the intrinsic states, are small. The states thus appear to be consistent with the Brink-Axel hypothesis, by which the same collective state (with the same width) is constructed on each of the intrinsic states. However, as the excitation energy of the intrinsic states increases, a corresponding increase in both the spreading width and the escape width of the state becomes observable, consistent with the increase in widths experimentally observed in hot giant resonances. Using a phenomenological expression for the compound escape width, $\Gamma_\nu \approx 14\exp(-4.69\sqrt{A/E^*})$ MeV and taking Sn as an example, we expect the total width to saturate at high excitation at $\Gamma_{1\nu} \approx 4.5$ MeV + 14 MeV, which compares fairly well with the observed value of 15 MeV [17].

The amplitude of the first excited collective state is obtained from a single action of the coupling V. Taking the final relative momentum of the projectile and target to be $\vec{k}'$, the amplitude $A_1(\vec{k},\vec{k}')$ for excitation of the first collective state is

$$A_1(\vec{k},\vec{k}') = \frac{1}{E - E_{\vec{k}'} - e_1 + i\,\Gamma_{10}/2} \left\langle 1,\varphi^{-}_{\vec{k}'}\right| V \left|\psi^{+}_{\vec{k}00}\right\rangle, \tag{9}$$

where $\left|\psi^{+}_{\vec{k}00}\right\rangle$ is the relative motion wavefunction in the entrance channel calculated by taking into account the coupling to the one-phonon excited state to all orders [8]. What is observed are the decay products of the excited state, which can decay either directly or after passing through the intrinsic states. The direct contribution is

$$d\sigma_1^{dir}(\vec{k},\vec{k}') = \Gamma^{\uparrow}_{10} \left|A_1(\vec{k},\vec{k}')\right|^2, \tag{10}$$

while the decay through the intrinsic states yields

$$d\sigma_1^{int}(\vec{k},\vec{k}') = \Gamma_{01}\frac{\Gamma^{\downarrow}_{10}}{\Gamma_{01}} \left|A_1(\vec{k},\vec{k}')\right|^2 = \Gamma^{\downarrow}_{10} \left|A_1(\vec{k},\vec{k}')\right|^2, \tag{11}$$

where Γ_{01} is the width of the intrinsic states at an energy $\varepsilon_\nu \approx e_1$.We have assumed, based on the complexity of the intrinsic states, that these possess no spreading width so that the width Γ_{01} is all escape width. Adding the two contributions, we have for the cross section

$$d\sigma_1(\vec{k},\vec{k}') = \frac{\Gamma_{10}}{(E - E_{\vec{k}'} - e_1)^2 + (\Gamma_{10}/2)^2} \left|\left\langle 1,\varphi^{-}_{\vec{k}'}\right| V \left|\psi^{+}_{\vec{k}00}\right\rangle\right|^2. \tag{12}$$

The second collective state can be populated predominantly through a two-step process. Assuming a final relative momentum of $\vec{k}''$, we have

$$A_2(\vec{k},\vec{k}'') = \frac{1}{E - E_{\vec{k}''} - e_2 + i\,\Gamma_{20}/2} \times \int d^3k' \left\langle 2,\varphi^{-}_{\vec{k}''}\right| V \left|1,\varphi^{+}_{\vec{k}'}\right\rangle \frac{1}{E - E_{\vec{k}'} - e_1 + i\,\Gamma_{10}/2} \left\langle 1,\varphi^{+}_{\vec{k}'}\right| V \left|\psi^{+}_{\vec{k}00}\right\rangle \tag{13}$$

This amplitude describes the process in which a second collective excitation occurs before the first collective state has decayed to the intrinsic states. The corresponding cross section is

$$d\sigma_2(\vec{k},\vec{k}'') = \frac{\Gamma_{20}}{(E-E_{\vec{k}''}-e_2)^2+(\Gamma_{20}/2)^2} \times \tag{14}$$
$$\left| \int d^3 k' \left\langle 2,\varphi^-_{\vec{k}''} \right| V \left| 1,\varphi^+_{\vec{k}'} \right\rangle \frac{1}{E-E_{\vec{k}'}-e_1+i\,\Gamma_{10}/2} \left\langle 1,\varphi^+_{\vec{k}'} \right| V \left| \psi^+_{\vec{k}00} \right\rangle \right|^2$$

For a harmonic mode, we expect $e_2 \approx 2\,e_1$.We also expect $\Gamma_{20} \approx 2\,\Gamma_{10}$, since we expect that $\langle 2\,|W\,|\,1\rangle \approx \sqrt{2}\,\langle 1\,|W\,|\,0\rangle$.

There is another two-step process – a fluctuating one – that can look as if it were an excitation of the second collective state, although in fact it is not. This is the process in which a second collective excitation occurs on top of the hot background of incoherent, intrinsic excitations remaining after the first collective state has decayed to the intrinsic states. The amplitude for this process, for an arbitrary intrinsic state $|\nu\rangle$, is

$$A^{\rm fl}_{2,\nu}(\vec{k},\vec{k}'') = \frac{1}{E-E_{\vec{k}''}-e_1-\varepsilon_\nu+i\Gamma_{1\nu}/2} \int d^3k' \left\langle 1\varphi^-_{\vec{k}''} \right| V \left| 0\varphi^+_{\vec{k}'} \right\rangle \times \tag{15}$$
$$\frac{1}{E-E_{\vec{k}'}-\varepsilon_\nu+i\Gamma_{0\nu}/2} \langle 0\nu\,|W|\,10\rangle \frac{1}{E-E_{\vec{k}'}-e_1+i\Gamma_{10}/2} \left\langle 1\varphi^+_{\vec{k}'} \right| V \left| \psi^+_{\vec{k}00} \right\rangle .$$

Its contribution to the cross section is incoherent with the others. When summed over the intrinsic states, it is

$$d\sigma^{\rm fl}_2(\vec{k},\vec{k}'') = \sum_\nu \left| A^{\rm fl}_{2,\nu}(\vec{k},\vec{k}'') \right|^2 . \tag{16}$$

To get even a crude estimate of the above expression, we have to do a bit of hand waving. To begin, we replace the intrinsic excitation energy ε_ν and the decay width $\Gamma_{1\nu}$ by their average values, $e_1$and Γ_{1c} in the final-state factor, so that

$$\frac{\Gamma_{1\nu}}{(E-E_{\vec{k}''}-e_1-\varepsilon_\nu)^2+(\Gamma_{1\nu}/2)^2} \longrightarrow \frac{\Gamma_{1c}}{(E-E_{\vec{k}''}-2\,e_1)^2+(\Gamma_{1c}/2)^2}, \tag{17}$$

and the factor can be removed from the sum. Next, we approximate the remaining sum over intrinsic states as

$$\sum_\nu \frac{|\langle 0,\nu\,|W\,|\,1,0\rangle|^2}{(E-E_{\vec{k}'}-\varepsilon_\nu+i\,\Gamma_{0\nu}/2)\,(E-E_{\vec{q}'}-\varepsilon_\nu-i\,\Gamma_{0\nu}/2)} \approx \frac{\Gamma^\downarrow_{10}}{\Gamma_{0c}} \frac{1}{1+i\,(E_{\vec{k}'}-E_{\vec{q}'})/\Gamma_{0c}}, \tag{18}$$

We have also replaced the intrinsic state decay widths $\Gamma_{0\nu}$ by their average value Γ_{0c} and have used $\vec{k}'$ and $\vec{q}'$ to denote the dummy variables of the two conjugate integrals. Finally, we argue that the restriction imposed on the momentum integrals by the right hand side of Eq.18 reduces them from their unrestricted value by a factor of Γ_{0c}/Γ_{10}. That is,

$$\int d^3k' \int d^3q' \frac{F(\vec{k}')\,F^*(\vec{q}')}{1+i\,(E_{\vec{k}'}-E_{\vec{q}'})/\Gamma_{0c}} \approx \frac{\Gamma_{0c}}{\Gamma_{10}} \left| \int d^3k'\,F(\vec{k}') \right|^2, \tag{19}$$

where $F(\vec{k}')$ is the rest of the integrand. With these three approximations, we can rewrite the fluctuation contribution to the cross section as

$$d\sigma^{\rm fl}_2(\vec{k},\vec{k}'') \approx \frac{\Gamma_{1c}}{(E-E_{\vec{k}''}-2\,e_1)^2+(\Gamma_{1c}/2)^2} \frac{\Gamma^\downarrow_{10}}{\Gamma_{10}} \times \tag{20}$$
$$\left| \int d^3 k' \left\langle 1,\varphi^-_{\vec{k}''} \right| V \left| 0,\varphi^+_{\vec{k}'} \right\rangle \frac{1}{E-E_{\vec{k}'}-e_1+i\,\Gamma_{10}/2} \left\langle 1,\varphi^+_{\vec{k}'} \right| V \left| \psi^+_{\vec{k}00} \right\rangle \right|^2 .$$

We can now compare the fluctuation contribution to the apparent excitation of the second collective state with the actual one of Eq.14. We first observe that both will have approximately the same average excitation energy of $2\,e_1$. The width of the fluctuation cross section, though, will be about half that of the real one, $\Gamma_{1c} \approx \Gamma_{10} \approx \Gamma_{20}/2$, since the contribution of the intrinsic states to the width is expected to be extremely small at these energies. Due to the difference in the matrix elements of V, the magnitude squared of the momentum integral of the fluctuation cross section will also be about half that of the actual one. The two cross sections will then be comparable at their peak values. The observed cross section, in these conditions, would be appreciably larger than the expected value and would have a width intermediate between the width Γ_{10} of the single giant resonance and the width $\Gamma_{20} \approx 2\,\Gamma_{10}$ expected for the double giant resonance.

Since the fluctuation contribution to the apparent excitation of the second collective state depends on the decay of the collective state into the intrinsic states, we would expect it to be important in a limited range of incident energy. At sufficiently high incident energies, we expect that the target would no longer have time to decay and be excited a second time by the projectile. This tendency can be seen in the experimental data for ^{208}Pb. The DGDR excitation cross section observed at a laboratory energy of about 100A MeV is a factor of two larger than the predicted one [5], while the cross section at 640A MeV is only about 30% greater than that calculated [6]. The observed width of the DGDR also tends to increase with the incident energy, consistent with the diminishing contribution of the fluctuation cross section.

An extension of the semiclassical model of ref. [8] to include the Brink-Axel effect will be reported elsewhere.

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[1] See eg. H. Emling, Prog. Part. Nucl. Phys. **33,** 729 (1994); Ph. Chomaz and N. Frascaria, Phys. Rep. **252**, 275 (1995).
[2] R. Schmidt *et al.*, Phys. Rev. Lett. **70**, 1767 (1993).
[3] T. Aumann *et al.*, Phys. Rev. C **47**, 1728 (1993).
[4] J.L. Ritman *et al.*, Phys. Rev. Lett. **70**, 533 (1993); **70**, 2659(E), (1993).
[5] J.R. Beene, Nucl. Phys. **A569**, 163c (1993).
[6] K. Boretzky *et al.*, Phys. Lett. B **384**, 30 (1996).
[7] Ph. Chomaz *et al.*, Nucl. Phys. **A318**, 41 (1994).
[8] L.F. Canto, A. Romanelli, M.S. Hussein, and A.F.R. de Toledo Piza, Phys. Rev. Lett. **72**, 2147 (1994); C.A. Bertulani, L.F. Canto, M.S. Hussein, and A.F.R. de Toldeo Piza, Phys. Rev. C **53**, 334 (1996).
[9] C. Volpe, F. Catara, Ph. Chomaz, M.V. Andres, and E.G. Lanza, Nucl. Phys. **A589**, 521 (1995).
[10] P.F. Bortignon and C.H. Dasso, Phys. Rev. C **56**, 574 (1997).
[11] C.A. Bertulani, V.Yu. Ponomarev, and V.V. Voronov, Phys. Lett. B **388**, 457 (1996).
[12] D. Brink, D. Phil. Thesis, Oxford University, Unpublished (1955); P. Axel, Phys. Rev. **126**, 671 (1962).
[13] C.M. Ko, Z.Phys. **A286**, 405 (1978).
[14] R.A. Broglia, C.H. Dasso, and Aa. Winther, Phys. Lett. B **48**, 302 (1974).
[15] D. Agassi, C. M. Ko and H. Weidenmüller, Ann. Phys. (N.Y.) **107**, 140 (1977).
[16] M. S. Hussein, M. P. Pato. and A. F. R. de Toledo Piza, Phys. Rev. **C51**, 846 (1995)
[17] J.J. Gaardhoje, Nucl. Phys. **A488**, 261c (1988).

FIGURE CAPTION

Figure 1: A schematic picture showing the coupling to the one "cold" phonon state (d), the two "cold" phonons state, the fine structure states of d (b) and the one "hot" Brink-Axel (B-A) phonon state.